

PATENT APPLICATION  
Navy Case No. 82,774

IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

## APPLICATION FOR LETTERS PATENT

TO ALL WHOM IT MAY CONCERN:

BE IT KNOWN THAT Michael L. Picciolo and Karl Gerlach who are citizens of the United States of America, and is a residents of, Silver Spring, MD and Dunkirk, MD, invented certain new and useful improvements in "PSEUDO-MEDIAN CASCADED CANCELLER" of which the following is a specification:

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PSEUDO-MEDIAN CASCADED CANCELLER

Field of the Invention

The invention relates in general to an adaptive signal processing system. More specifically, the invention relates to an adaptive signal processing system that utilizes a pseudo-median cascaded canceller to calculate complex adaptive weights and filter out undesired noise.

Background of the Invention

Adaptive signal processing systems have many applications including radar reception, cellular telephones, communications systems, and biomedical imaging. Adaptive signal processing systems utilize adaptive filtering to differentiate between the desired signal and noise. An adaptive filter is defined by four aspects: the type of signals being processed, the structure that defines how the output signal of the filter is computed from its input signal, the parameters within this structure that can be iteratively changed to alter the filter's input-output relationship, and the adaptive algorithm that describes how the parameters are adjusted from one time instant to the next.

Common applications of adaptive signal processing include: an adaptive radar reception antenna array, an adaptive antenna array for adaptive communications, and adaptive sonar. In these systems, desired signal detection and estimation is hindered by noise and interference. Interference may be intentional jamming and or unintentional received radiation. These antenna arrays may change their reception patterns automatically in response to the signal environment in a way that optimizes the signal power to interference power plus noise power ratio (abbreviated as SINR). The array pattern is

1 easily controlled by weighting the amplitude and phase of the signal from each element  
2 before combining (adding) the signals. Adaptive arrays are especially useful to protect  
3 radar and communication systems from interference when the directions of the interference  
4 are unknown or changing while attempting to receive a desired signal of known form.  
5 Adaptive arrays are capable of operating even when the antenna elements have arbitrary  
6 patterns, polarizations, and spacings. This feature is especially advantageous when an  
7 antenna array operates on an irregularly shaped surface such as an aircraft or ship.

8 Adaptive signal processing systems are required to filter out undesirable noise. Due  
9 to the lack of a priori knowledge of an external environment, adaptive signal processing  
10 systems require a certain amount of statistically independent weight training data samples  
11 to effectively estimate the input noise covariance matrix. The input noise covariance  
12 matrix is the set of second central moments (i.e., moments about the mean) among the  
13 received antenna element voltages.

14 "Ideal" weight training data has a Gaussian probability distribution for both its real  
15 and imaginary baseband components. Fig. 1 is a frequency vs. amplitude plot representing  
16 an ideal Gaussian noise source with a bell-shaped probability curve in which the mean  
17 value is identical to the median value. However, real-world weight training data may be  
18 contaminated by undesirable impulse noise outliers, resulting in a non-Gaussian  
19 distribution of real and imaginary components.

20 The number of weight training data samples required for signal power to  
21 interference power plus noise power ratio (SINR) performance of the adaptive processor to  
22 be within 3dB of the optimum on average is called the convergence measure of  
23 effectiveness (MOE) of the processor. A signal is stationary if its statistical probability

1 distribution is independent of time. For the pure statistically stationary Gaussian noise  
2 case, the convergence MOE of the conventional Sample Matrix Inversion (SMI) adaptive  
3 linear technique can be attained using approximately  $2N$  samples for adaptive weight  
4 estimation, regardless of the input noise covariance matrix, where  $N$  is the number of  
5 degrees of freedom in the processor (i.e., the number of antenna elements or subarrays) for  
6 a spatially adaptive array processor, or  $N$  is the number of space-time channels in a space-  
7 time adaptive processing (STAP) processor). Referred to as the SMI convergence MOE,  
8 convergence within 3dB of the optimum using approximately  $2N$  samples for adaptive  
9 weight estimation has become the benchmark used to assess convergence rates of adaptive  
10 processors. General information regarding SMI convergence MOE may be found in Reed,  
11 I.S., Mallet, J.D., Brennan, L.E., "Rapid Convergence Rate in Adaptive Arrays", IEEE  
12 Trans. Aerospace and Electronic Systems, Vol. AES-10, No. 6, November, 1974, pp. 853-  
13 863, the disclosure of which is incorporated herein by reference.

14 Conventional sample matrix inversion (SMI) adaptive signal processing systems  
15 are capable of meeting this benchmark for the pure statistically stationary Gaussian noise  
16 case. If, however, the weight training data contains non-Gaussian noise outliers, the  
17 convergence MOE of the system increases to require an unworkably large number of  
18 weight training data samples. The performance degradation of the SMI algorithm in the  
19 presence of non-Gaussian distributions (outliers) can be attributed to the highly sensitive  
20 nature of input noise covariance matrix estimates to even small amounts of impulsive non-  
21 Gaussian noise that may be corrupting the dominant Gaussian noise distribution. General  
22 information regarding the sensitivity of the SMI algorithm may be found in Antonik, P.  
23 Schuman, H. Melvin, W., Wicks, M., "Implementation of Knowledge-Based Control for

1 Space-Time Adaptive Processing", IEEE Radar 97 Conference, 14-16 Oct., 1997, p. 478-  
2 482, the disclosure of which is incorporated herein by reference.

3 Thus, for contaminated weight training data, convergence rate slows significantly  
4 with conventional systems. Fast convergence rates are important for several practical  
5 reasons including limited amounts of weight training data due to non-stationary noise and  
6 computational complexity involved in generating adaptive weights. In other words, the  
7 time which elapses while a conventional system is acquiring weight training data and  
8 generating adaptive weights may exceed the duration of a given non-stationary noise  
9 environment, and an adaptive weight thus generated has become obsolete prior to  
10 completion of its computation.

11 Most real world data does not have a purely Gaussian probability distribution due  
12 to contamination by non-Gaussian outliers. Conventional signal processors assume that  
13 the weight training data has a Gaussian distribution, and therefore they do not perform as  
14 well as theory would predict when operating with real world data. If weight training data  
15 contains desired signals that appear to be outliers, the performance is similarly degraded.  
16 In an effort to compensate for these performance problems, conventional systems employ  
17 subjective data screening techniques to remove perceived outliers from the data prior to  
18 processing. However, subjective screening is undesirable because the process is ad-hoc in  
19 nature, requires many extra processing steps, and may even degrade system performance.

20 It would therefore be desirable to provide an adaptive signal processing system that  
21 accommodates outlier contaminated weight training data and still produces a convergence  
22 MOE which is comparable to the above referenced benchmark for SMI systems. It would  
23 also be desirable to provide a system that significantly reduces desired signal cancellation

1 when weight training data includes desired signal components. See generally Gerlach, K.,  
2 Kretschmer, F. F., Jr., "Convergence Properties of Gram-Schmidt and SMI Adaptive  
3 Algorithms, Part II", IEEE Trans. Aerospace and Electronics Systems, Vol. 27, No. 1, Jan.  
4 1991, pp. 83-91, the disclosure of which is incorporated herein by reference.

5  
6 Summary of the Invention

7 The present invention provides an adaptive signal processing system that utilizes a  
8 pseudo-median cascaded canceller to compute a complex adaptive weight and generate a  
9 filtered output signal. The effect of non-Gaussian noise contamination on the convergence  
10 MOE of the system is negligible. In addition, when desired signal components are  
11 included in weight training data they cause negligible loss of noise cancellation.

12 In a preferred embodiment, the system and method receive a plurality of input  
13 signals corresponding to a common target signal and sequentially decorrelate the input  
14 signals to cancel the correlated noise components therefrom. The system includes a  
15 plurality of building blocks arranged in a Gram-Schmidt type cascaded canceller  
16 configuration for sequentially decorrelating each of the input signals from each other of the  
17 input signals to thereby yield a single filtered output signal. Each building block includes  
18 a local main input channel which receives a main input signal, a local auxiliary input  
19 channel which receives an auxiliary input signal, and a local output channel which sends a  
20 filtered output signal. Each building block generates a complex adaptive weight whose  
21 real and imaginary parts are found by taking the sample median value of the ratio of local  
22 main input weight training data to local auxiliary input weight training data for the real and

1 imaginary parts separately. That is, each building block generates a complex adaptive  
2 weight,  $w_{med}$ , by solving the equation:

$$w_{med} = MED_{k=1 \text{ to } K} \left[ \text{real} \left( \frac{z(k)^*}{x(k)^*} \right) \right] + j \left\{ MED_{k=1 \text{ to } K} \left[ \text{imag} \left( \frac{z(k)^*}{x(k)^*} \right) \right] \right\}$$

4 where K is the number of weight training data samples, z is the local main input signal,  
5 \* means "complex conjugate", and x is the local auxiliary input signal. The number of  
6 weight training data samples K should be odd for a true median value. K may be even, but  
7 if so, then MED refers to a numerical approximation to the median such as one or the other  
8 of two center marked values (or to their average). Each building block then uses the  
9 complex adaptive weight to generate a local output signal r by solving the equation:

$$r = z - w_{med}^* x, \quad (\text{where } w_{med}^* \text{ is multiplied by } x).$$

11 Other advantages and features of the invention will become apparent from the  
12 following detailed description of the preferred embodiments and the accompanying  
13 drawings.

#### 15 Brief Description of the Drawings

16 The invention will now be described with reference to certain preferred  
17 embodiments thereof and the accompanying drawings, wherein:

18 Fig. 1 illustrates a Gaussian noise distribution;

19 Fig. 2 is a block diagram of a general linear adaptive array;

20 Fig. 3 is a block diagram of a conventional sample matrix inversion cascaded  
21 processor;

Fig. 4 is a single block of a conventional sample matrix inversion cascaded processor;

Fig. 5 is a graph of the probability density function of real or imaginary parts of  $s = (r_u^o/x^o)^*$ , where  $x^o$  and  $r_u^o$  are normalized versions of input variables  $x$  and output variable  $r_u$ , each with unit variance.

Fig. 6 is a block diagram of a pseudo-median cascaded canceller processor in accordance with the present invention;

Fig. 7 is a single building block of a pseudo-median cascaded canceller processor in accordance with the present invention;

Fig. 8 is a graph showing the convergence rate for different levels of non-Gaussian noise for a conventional sample matrix inversion processor which plots normalized output power residue vs. number of samples.

Fig. 9 is a graph showing the convergence rate for different levels of non-Gaussian noise for a sample matrix inversion processor in accordance with the present invention which plots normalized output power residue vs. number of samples, and

Fig. 10 is a block diagram of a pseudo-median cascaded canceller processor in accordance with the present invention illustrating another aspect of the invention.

#### Detailed Description of the Preferred Embodiments

Initially, a conventional general adaptive array having a sidelobe canceller form and a conventional Gram-Schmidt side lobe canceller will be described. The adaptive implementation of a conventional Gram-Schmidt cascaded canceller provides a cascaded set of operationally identical, two-input canceller Gram-Schmidt  $L_2$  building blocks. In

1 accordance with the pseudo-median cascaded canceller of present invention, these Gram-  
2 Schmidt  $L_2$  building blocks are replaced with new pseudo-median  $L_{med}$  building blocks.  
3 Each pseudo-median  $L_{med}$  building block generates a complex adaptive weight whose real  
4 and imaginary parts are found by taking the sample median value of the ratio of local main  
5 input weight training data to local auxiliary input weight training data for the real and  
6 imaginary parts separately, applies the computed weight to a function of a local main input  
7 signal and a local auxiliary input signal, and generates a filtered output signal.  
8 Theoretically, the pseudo-median  $L_{med}$  building blocks produce the same optimal weight as  
9 the Gram-Schmidt  $L_2$  building blocks if the number of samples of training data is very  
10 large ( $K \rightarrow \infty$ ), and the two inputs each have symmetric probability density functions about  
11 the mean for their real and imaginary components (an example is the Gaussian pdf). In  
12 addition, the convergence MOE of the pseudo-median cascaded canceller offers a  
13 significant improvement upon Gram-Schmidt cascaded cancellers in the presence of non-  
14 Gaussian noise outliers or equivalently, for some cases, when desired signal components  
15 are present in the training data.

16 Fig. 2 is a block diagram of a conventional general linear adaptive array processor  
17 10. In Fig. 2,  $\mathbf{q}$  is the input data vector,  $y$  is the output scalar,  $\mathbf{w}$  is the complex adaptive  
18 weight vector, and  $H$  denotes Hermitian or conjugate-transpose operation. The general  
19 linear adaptive array processor 10 with a single mainbeam steering vector constraint may  
20 be converted to its equivalent sidelobe canceller form via a unitary, non-singular, square  
21 matrix transformation of the input data vector  $\mathbf{q}$ . See generally Haykin, S., *Adaptive Filter*  
22 *Theory*, 3<sup>rd</sup> Ed., Prentice-Hall, New Jersey, 1996; Gerlach, K., Kretschmer, F. F., Jr.,  
23 "Convergence Properties of Gram-Schmidt and SMI Adaptive Algorithms", IEEE Trans.

1 Aerospace and Electronics Systems, Vol. 26, No. 1, Jan. 1990, pp. 44-56; and Brennan,  
2 L.E., Reed, I.S., "Digital Adaptive Arrays With Weights Computed From And Applied To  
3 The Same Sample Set", Proceedings 1980 Adaptive Antenna Symposium, Rome Air  
4 Development Center report RADC-TR-80-378, Vol. 1, Dec. 1980, pp. 236-249, the  
5 disclosure of which are incorporated herein by reference.

6 A non-singular, matrix transformation of the inputs of a general adaptive array does  
7 not change the theoretical SINR if weight adaptation using SMI is subsequently carried out  
8 using an identically transformed steering vector. Thus, a linear transformation can be  
9 chosen that moves all desired signal energy into a single "main" channel (as well as some  
10 noise), leaving only (transformed) noise in the remaining "auxiliary" channels, creating a  
11 sidelobe canceller form,  $\mathbf{u}$ , of the input data vector,  $\mathbf{q}$ . The first element or "main channel"  
12 of  $\mathbf{u}$  is labeled  $\mathbf{u}_m = u_1$  and "auxiliary channels" are labeled  $\mathbf{u}_a = u_n$ , for  $n=2, \dots, N$ .

13 A sidelobe canceller is a specific form of the general adaptive array where the  $N \times$   
14 1 desired signal (steering) vector is set to  $[1 \ 0 \ 0 \ \dots \ 0]^T$ , the "main" or first channel weight  
15 is fixed to unity, and the  $(N-1) \times 1$  auxiliary channel adaptive weight vector estimate  $\hat{\mathbf{w}}_a$   
16 (^ denotes 'estimate of') solves the adaptive Wiener-Hopf matrix equation,

$$\hat{\mathbf{w}}_a = \hat{\mathbf{R}}_a^{-1} \hat{\mathbf{r}}_{am}, \quad (1)$$

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18  
19  
20 where  $\hat{\mathbf{R}}_a$  is the  $(N-1) \times (N-1)$  auxiliary channel input noise covariance matrix estimate,  
21 and  $\hat{\mathbf{r}}_{am}$  is the  $(N-1) \times 1$  cross-correlation vector estimate between the auxiliary channels

1 and the main channel. For Gaussian statistics, the maximum likelihood estimates of these  
2 quantities are

$$\hat{\mathbf{R}}_a = \frac{1}{K} \sum_{k=1}^K \mathbf{u}_a(k) \mathbf{u}_a(k)^H \quad (2)$$

5 and

$$\hat{r}_{am} = \frac{1}{K} \sum_{k=1}^K \mathbf{u}_a(k) u_m(k)^* , \quad (3)$$

7  
8 where K is the number of weight training data samples used in the averages and \* denotes  
9 complex conjugate operation. When employing equation (2) and equation (3), the sidelobe  
10 canceller uses the SMI algorithm, and its scalar output, g, is (using vector partitioning)

$$g = \mathbf{w}_{slc}^H \mathbf{u} = \begin{bmatrix} 1 \\ \dots \\ -\hat{\mathbf{w}}_a \end{bmatrix}^H \mathbf{u} . \quad (4)$$

11  
12  
13 The conventional SMI-based sidelobe canceller in equation (4) has an equivalent  
14 Gram-Schmidt cascaded canceller form in the steady state and in the transient state with  
15 numerically identical outputs, where transient state refers to the case where weights are  
16 estimated from finite length data vectors. Infinite numerical accuracy is assumed, i.e., the  
17 cascaded weights correspond to a numerically equivalent set of linear weights equation (1)  
18 that can be applied directly to the transformed array input data vector  $\mathbf{u}$  via equation (4), if  
19 so desired.

Fig. 3 illustrates a conventional Gram-Schmidt canceller 12 for the  $N = 4$  channel case. See generally, Monzingo, R.A., Miller, T.W, *Introduction to Adaptive Arrays*, John Wiley and Sons, New York, 1980, the disclosure of which is incorporated herein by reference. It is comprised of six identical two-input canceller Gram-Schmidt  $L_2$  building blocks 14. In each building block 14, the optimal weight estimate for that building block is calculated by minimizing the square of the  $L_2$ -norm of the residual output vector from that building block, over some specified number of  $K$  weight training data samples.

Fig. 4 illustrates a single Gram-Schmidt  $L_2$  building block 14. For notational simplicity, the left input of any single building block is relabeled  $z$ , the right input is relabeled  $x$ , and the output is relabeled  $r$ . Each building block serves to have the component of  $z$ , which is correlated with  $x$ , subtracted from  $z$ . This is accomplished by choosing an optimum weight estimate  $\hat{w}_{opt}$  such that the residual  $r$  is statistically uncorrelated to  $x$ . The least-squares method is used to estimate  $w_{opt}$  by minimizing over the set of complex weights  $w$ , the square of the  $L_2$  norm of the residual output  $r = z - w^* x$  where  $r = r(k)$ ,  $z = z(k)$ , and  $x = x(k)$ , for  $k = 1, \dots, K$ ; i.e., for any single building block,

$$\hat{w}_{opt} = \arg \min_w \left[ \frac{1}{K} \sum_{k=1}^K |z(k) - w^* x(k)|^2 \right] . \quad (5)$$

This results in the scalar adaptive Wiener-Hopf equation,

$$\hat{w}_{opt} = \hat{R}_{xx}^{-1} \hat{r}_{xz} , \quad (6)$$

where maximum likelihood estimates again may be used for the scalar input noise covariance estimates, assuming Gaussian statistics, as

$$\hat{R}_{xx} = \frac{1}{K} \sum_{k=1}^K x(k)x(k)^* , \quad (7)$$

and

$$\hat{r}_{xz} = \frac{1}{K} \sum_{k=1}^K x(k)z(k)^* . \quad (8)$$

Note that equation (7) and equation (8) are sensitive to non-Gaussian noise contaminants just like their matrix counterparts in equation (2) and equation (3).

As set forth above the conventional general adaptive array processor may be equivalently transformed in terms of SINR into its conventional Gram-Schmidt cascaded canceller form. The adaptive implementation of a conventional Gram-Schmidt cascaded canceller provides a cascaded set of operationally identical, two-input canceller  $L_2$  building blocks.

Replacing the  $L_2$  building blocks with pseudo-median  $L_{med}$  building blocks throughout a cascaded canceller results in the pseudo-median cascaded canceller configuration of the present invention shown in Fig. 6. Fig. 7 illustrates a single pseudo-median  $L_{med}$  building block canceller. Each of the pseudo-median  $L_{med}$  building blocks computes a complex adaptive weight  $w_{med}$  whose real and imaginary parts are found by taking the sample median value of the ratio of local main input weight training data to

1 local auxiliary input weight training data for the real and imaginary parts separately. Each  
2 of the pseudo-median  $L_{med}$  building blocks 18 then applies the complex conjugate of the  
3 computed weight  $w_{med}^*$  to a function of a local main input signal and a local auxiliary input  
4 signal, and generates a filtered local output signal,  $r$ , by solving the following equation:  $r =$   
5  $z - w_{med}^* x$ . Note that Fig. 6 illustrates pseudo-median cascaded canceller with  $N=4$   
6 inputs, however, any number  $N$  of inputs may be utilized.

7 In the first level of processing,  $N$  input signals are input into  $N-1$  pseudo-median  
8  $L_{med}$  building blocks 18 to generate  $N-1$  local output signals. In the next level of  
9 processing,  $N-1$  local input signals are input into  $N-2$  pseudo-median  $L_{med}$  building blocks  
10 18 to generate  $N-2$  local output signals. This process is repeated until one final output  
11 signal remains.

12 The pseudo-median cascaded canceller of the present invention may be  
13 implemented by a set of program instructions on an arithmetical processing device such as  
14 a general-purpose digital signal processor (DSP) or microprocessor to carry out in real time  
15 the computational steps presented above. Alternatively, custom-built application specific  
16 integrated circuits (ASIC), field-programmable gate arrays (FPGA), or firmware can be  
17 fabricated to perform the same computations as a set of logic instructions in hardware.  
18 These implementations are interchangeable.

19 The pseudo-median  $L_{med}$  building block 18 computes complex adaptive weight,  $w_k$ ,  
20 as the sum of a real part and an imaginary part, which are found by taking the sample  
21 median value of the ratio of local main input weight training data to local auxiliary input  
22 weight training data for the real and imaginary parts separately. First, form the set  $w_k$  as  
23  $w_k = (z(k)/x(k))^*$ , for  $k = 1, 2, \dots, K$  where  $K$  is the number of training samples. The sample

1 median of the real parts of  $\{w_k\}$  is taken as the real part of the new optimal weight, and the  
2 sample median of the imaginary parts of  $\{w_k\}$  is taken as the imaginary part of the new  
3 optimal weight. As  $K \rightarrow \infty$ , and assuming that, for this analysis, there are no outliers, the  
4 resulting adaptive weight,

$$w_{med} = \underset{k=1 \text{ to } K}{MED} \left[ \text{real} \left( \frac{z(k)^*}{x(k)^*} \right) \right] + j \left\{ \underset{k=1 \text{ to } K}{MED} \left[ \text{imag} \left( \frac{z(k)^*}{x(k)^*} \right) \right] \right\} \quad (9)$$

5  
6  
7  
8 converges to the same optimal complex weight as a Gram-Schmidt  $L_2$  building block using  
9 the same weight training data. This convergence occurs if both  $z$  and  $x$  each have a zero  
10 mean, Gaussian probability density function (pdf) for both their real and imaginary parts,  
11 and even more generally, convergence occurs if all four pdf's are symmetric (i.e.,  $z$  and  $x$   
12 each have complex symmetric densities). In equation (9), MED refers to the sample  
13 median (or, if  $K$  is even, the MED may be taken as either of the middle two order-ranked  
14 samples, or their average),  $j = \sqrt{-1}$  is the unit imaginary number, real means "real part of",  
15 and imag means "imaginary part of".

16 Note that alternative embodiments of the invention allow the processor to calculate  
17 and use only the real or only the imaginary portion of  $w_{med}$ .

18 For the following description, the explicit  $k$  dependence is dropped from the  
19 variables  $z$ ,  $x$ ,  $r$ , and  $w$ , for notational simplicity, and  $K$  is assumed to approach infinity  
20 ( $K \rightarrow \infty$ ). Starting with the Gram-Schmidt  $L_2$  building block canceller, if a priori  
21 knowledge of the scalar auxiliary channel input noise covariance  $R_{xx}$  and scalar cross-  
22 correlation  $r_{xz}$  are available, the optimal weight is obtained using mean square error

(MSE) criterion and results in the two-input, scalar, Wiener-Hopf equation,  $w_{opt} = R_{xx}^{-1} r_{xz}$ .

The residual value output is defined as  $r_u = z - w_{opt}^* x$  after optimal weighting is applied, and

$r_u$  is uncorrelated with  $x$  by definition. Solving for  $z$  yields  $z = r_u + w_{opt}^* x$ , and substituting

this into  $w = (z/x)^*$  from equation (9), results in

$$w = \left( \frac{z}{x} \right)^* = \frac{r_u^* + w_{opt}^* x^*}{x^*} = \left( \frac{r_u}{x} \right)^* + w_{opt} \quad (10)$$

Taking, separately, the statistical medians of the real and imaginary parts of equation (10) yields

$$w_{med_{r,i}} = med_{r,i} \left( \left( \frac{r_u}{x} \right)^* + w_{opt} \right) = med_{r,i} \left[ \left( \frac{r_u}{x} \right)^* \right] + w_{opt_{r,i}} \quad (11)$$

where  $med_{r,i}$  refers to the statistical median of both of the real and imaginary parts separately, and similarly for  $w_{opt_{r,i}}$ ; since  $w_{opt_{r,i}}$  are constants, they come out of the median function.

For the case where  $z$  and  $x$  are each zero mean, complex Gaussian random variables,  $med_{r,i}[(r_u/x)^*] = 0$ , yielding the desired components of the optimal weight,

$$w_{opt} = w_{opt_r} + jw_{opt_i}.$$

The random quantity  $r_u$  is a linear combination of  $z$  and  $x$  and is therefore a zero mean, complex Gaussian random variable. Since  $r_u$  and  $x$  are uncorrelated and Gaussian,

they are independent. The quotient  $(r_u/x)^*$  is normalized by multiplying by the ratio of standard deviations,

$$s = \frac{\sigma_x}{\sigma_{r_u}} \left( \frac{r_u}{x} \right)^* = \left( \frac{r_u^\circ}{x^\circ} \right)^* , \quad (12)$$

where  $\sigma_x$  and  $\sigma_{r_u}$  are the standard deviations of  $x$  and  $r_u$ , respectively, and  $x^\circ$  and  $r_u^\circ$  are normalized versions of  $x$  and  $r_u$ , respectively, each with unit variance. The pdf and cumulative distribution function (cdf) of  $s_{r,i}$  (subscripts  $r$  and  $i$  refer to the real and imaginary parts of  $s$ , separately) are respectively derived as

$$f_{s_{r,i}}(s_{r,i}) = \frac{1}{2(s_{r,i}^2 + 1)^{3/2}} , \quad -\infty \leq s_{r,i} \leq \infty , \quad (13)$$

and

$$F_{s_{r,i}}(s_{r,i}) = \frac{1}{2} + \frac{s_{r,i}}{2(s_{r,i}^2 + 1)^{1/2}} , \quad -\infty \leq s_{r,i} \leq \infty . \quad (14)$$

Fig. 5 is a graph of the probability density function of the real or imaginary parts of  $s = (r_u^\circ/x^\circ)^*$ . The terms  $f_{s_{r,i}}(s_{r,i})$  and  $s_{r,i}$  refer separately to the real,  $r$ , and imaginary,  $i$ , parts of  $f_s(s)$  and  $s$ . It is readily seen in Fig. 6 and via equation (13) that  $med_{r,i}(s) = 0$ . Using the fact that  $med_{r,i}(ay) = a med_{r,i}(y)$  for any real constant  $a$  and any complex random variable  $y$ , equation (11) becomes via equation (12),

$$w_{med_{r,i}} = w_{opt_{r,i}} \quad (15)$$

Therefore  $w_{med} \rightarrow w_{opt}$  as  $K \rightarrow \infty$ , as claimed for equation (9).

The convergence of a pseudo-median  $L_{med}$  building block and convergence performance of  $w_{med}$  will now be described. For Gaussian statistics, and for adaptive implementations of the two-input pseudo-median cascaded canceller (i.e., for finite K), the order statistics pdf,  $f_p(s_{r,i})$  is defined as a function of  $s_{r,i}$  as

$$f_p(s_{r,i}) = \frac{K!}{(p-1)!(K-p)!} F_{s_{r,i}}^{p-1}(s_{r,i}) \times [1 - F_{s_{r,i}}(s_{r,i})]^{K-p} f_{s_{r,i}}(s_{r,i}) \quad (16)$$

This is used to determine the median order statistics of  $s_{r,i}$ , using equations (13) and (14), where  $p = (K+1)/2$  is the median (for K odd). For convenience, just the results will be presented: using equation (12), the means of the median order statistics were found to be zero, and the variances of the median order statistics were found to be:  $\text{var}(med_{r,i}(s)) = 1/(K-1)$  (where 'var' denotes variance), for K an odd integer and  $K > 1$ . Thus, because  $\text{var}(ay_r) = a^2 \text{var}(y_r)$  holds for any real constant  $a$  and for any real random variable  $y_r$ , the variance of  $w_{med_{r,i}}$  equation (11) is

$$\text{var}(w_{med_{r,i}}) = \frac{\sigma_{r_u}^2}{\sigma_x^2} \left( \frac{1}{K-1} \right) \quad (17)$$

This variance will be used in the following derivation of the analytical convergence rate.

The figure of merit often used to assess canceller performance is the normalized output residue power (NORP), here labeled  $\eta$ , and it is approximately equal to the inverse of the SINR performance metric for non-concurrent processing. Non-concurrent processing refers to the case where adaptive weights are trained using secondary weight training data, but are applied to statistically independent primary data possibly containing desired signal components. This provides justification for directly comparing the SINR convergence MOE to the NORP convergence MOE since desired signal, which is only in the primary data (and only in the main channel), is assumed to pass through the canceller unaffected. For a two-input canceller,  $\eta$  is defined as,

$$\eta = \frac{E\{|z - w_o^* x|^2\}}{res_{opt}} \quad , \quad (18)$$

where  $w_o$  is the weight chosen under some arbitrary performance criterion,  $E$  denotes expectation, and

$$res_{opt} = E\{|z - w_{opt}^* x|^2\} \quad (19)$$

is the optimal minimum residue power found by using  $w_{opt} = R_{xx}^{-1} r_{xz}$ . If  $w_o$  in equation (18) is chosen under MSE criterion, then  $w_o = w_{opt}$ , and  $\eta$  achieves its minimum value of 1, or equivalently, 0 dB. However, this requires perfect a priori knowledge of the relevant statistics, so for adaptive methods such as SMI or pseudo-median  $L_{med}$  criterion,

$$w_o = w_{opt} + \tilde{w} \quad , \quad (20)$$

where  $\tilde{w}$  is some difference weight from the complex constant  $w_{opt}$ . Since  $r_u = z - w_{opt}^* x$ , using equation (18) and equation (20):

$$\eta = \frac{E\{|z - w_{opt}^* x - \tilde{w}^* x|^2\}}{res_{opt}} = \frac{E\{|r_u - \tilde{w}^* x|^2\}}{res_{opt}} \quad \text{and,} \quad (21)$$

and,

$$res_{opt} = E\{|z - w_{opt}^* x|^2\} = E\{|r_u|^2\} = \sigma_{r_u}^2 \quad . \quad (22)$$

Since  $r_u$  is uncorrelated with  $x$  by definition,  $E\{r_u x^*\} = E\{x r_u^*\} = 0$ , so, from the numerator of equation (21),

$$E\{|r_u - \tilde{w}^* x|^2\} = \sigma_{r_u}^2 + |\tilde{w}|^2 \sigma_x^2 \quad , \quad (23)$$

thus, substituting equation (22) and equation (23) into equation (21), results in

$$\eta = 1 + |\tilde{w}|^2 \frac{\sigma_x^2}{\sigma_{r_u}^2} = 1 + \tilde{w}_r^2 \frac{\sigma_x^2}{\sigma_{r_u}^2} + \tilde{w}_i^2 \frac{\sigma_x^2}{\sigma_{r_u}^2} \quad . \quad (24)$$

Note that subscripts  $r$  and  $i$  refer to the real and imaginary components of  $\tilde{w}$ , respectively.

The quantity  $w_{\text{med}}$  in equation (9) was shown to converge to  $w_{\text{opt}}$ , and its variance about the mean of  $w_{\text{opt}}$ , as a function of  $K$ , was given in equation (17) for both the real and imaginary parts of the weights. For the two-input pseudo-median cascaded canceller,  $\tilde{w}_{r,i} = w_{\text{med } r,i} - w_{\text{opt } r,i}$ , so  $E\{\tilde{w}_r^2\} = E\{\tilde{w}_i^2\} = \text{var}(w_{\text{med } r,i}) = \sigma_{r_u}^2 / [\sigma_x^2(1/(K-1))]$ . Thus, from equation (24), the average convergence rate for the pseudo-median  $L_{\text{med}}$  building block is found to be

$$E\{\eta\} = 1 + 2/(K-1) \quad , \quad (25)$$

for zero mean, complex Gaussian inputs ( $z$  and  $x$ ). In comparison, the conventional Gram-Schmidt  $L_2$  building block converges just slightly faster:  $1+1/(K-1)$ , for the same assumptions, but, as will be shown in the next section, it is not nearly as robust as the pseudo-median  $L_{\text{med}}$  building block. Lastly, it is noted that the two-input pseudo-median  $L_{\text{med}}$  algorithm just derived, has, like the two input SMI ( $L_2$ ) algorithm, a convergence rate shown here to be only a function of the number of samples,  $K$ , and thus is independent of the two-input, input noise covariance matrix.

The performance of the conventional SMI and pseudo-median cascaded canceller of the present invention were compared in the presence of a single sample noise outlier in the weight training data. The noise was given a range of powers, normalized to the internal noise level. The noise outlier was restricted to be in the weight training data only. Adapted weights were not applied to the same data that was used to train the adapted weights. Instead, they were applied to statistically independent data (non-concurrent

1 processing was used). For a canceller configuration, the desired signal in the main channel  
2 is passed through to the output with unity gain while only correlated noise is removed at  
3 each stage.

4 For simulations discussed next and shown in Figs 8 and 9, noise outliers were only  
5 added to the main channel ( $u_1$ ) in the weight training data, emulating the addition of a  
6 scaled desired-signal vector,  $[a \ 0 \ \dots \ 0]^T$ , where  $a^2$  is the noise outlier or added desired  
7 signal power. The addition of noise outliers to all channels, or to just the auxiliary  
8 channels only, resulted in much less degradation of the SMI convergence rate for all noise  
9 power levels (results not shown.) It will be seen for the pseudo-median cascaded canceller  
10 of the present invention that desired signal components present in the main channel of the  
11 weight training data have a significantly reduced effect on noise cancellation than in the  
12 conventional SMI ( $L_2$ ) cascaded processor.

13 Figures 8 and 9 will illustrate the advantages of the present invention.

14 Convergence plots for a conventional SMI ( $L_2$ ) cascaded canceller are shown in  
15 Fig. 8 for various noise powers. The value for  $\eta_{avg}$ , a Monte Carlo average of 20  
16 normalized output residue powers, is plotted vs. K, the number of weight training samples  
17 used. Ten channels were chosen ( $N = 10$ ), and one +20dB narrowband Gaussian noise  
18 barrage sidelobe jammer (20dB above internal receiver noise power) plus uncorrelated  
19 Gaussian noise were modeled as inputs in the simulations shown here. The SMI algorithm  
20 predicts 3dB convergence in about  $K = 2N = 20$  samples, which appears to be satisfied for  
21 plots corresponding to negligible noise outlier power values (-10dB to +10dB). However,  
22 as the noise outlier power increases, it is evident that convergence slows significantly. For  
23 example, for a single +20dB noise outlier, the convergence MOE is about 27 samples; for a

1 single +30dB noise outlier, many more than 50 samples are required. For three +20dB  
2 noise outliers (equal to the jammer level and therefore difficult to prescreen) (graph not  
3 shown here) the convergence MOE is 48 samples. The curves in Figure 8 thus illustrate  
4 the degradation in performance of a conventional SMI ( $L_2$ ) processor when applied to data  
5 with increasing noise outlier powers.

6 As shown in Fig. 9, for the pseudo-median cascaded canceller of the present  
7 invention, also with  $N=10$  input channels, convergence is essentially unaffected by the  
8 addition of noise outliers of any power level. The convergence rate of the present  
9 invention is approximately equal to the ideal SMI convergence rate in pure Gaussian  
10 jammer and noise environments. In fact, for three and even five noise outliers of any  
11 power level (graphs not shown here), convergence is still essentially unaffected. Thus, it is  
12 evident that strong desired signals in the weight training data cause little loss in noise  
13 cancellation; the pseudo-median cascaded canceller equivalent adaptive weight vector  
14 quickly approaches the optimum weight vector and the median function essentially ignores  
15 the added desired signal vector(s).

16 It is well known that for the SMI algorithm in Gaussian noise with no noise  
17 outliers, the convergence rate is independent of the input noise covariance matrix for both  
18 the two-input AND general  $N$ -input cases.

19 For the pseudo-median cascaded canceller, however, for the same assumptions, it  
20 appears that strict invariance to the input noise covariance matrix is generally limited to  
21 just the two-input case. However, simulations (not shown here) indicate that for more than  
22 two inputs, this desired invariance to the input noise covariance matrix is, in fact, true  
23 when the number of discrete noise sources is approximately one-third or less of the total

1 number of degrees of freedom  $N$ . This situation is representative of many realistic (low  
2 rank) noise scenarios, making the pseudo-median canceller of the present invention very  
3 attractive for many real-world processing environments. Even as the number of noise  
4 sources increases greater than this threshold, the convergence rate degrades gracefully and  
5 eventually becomes similar to the SMI ( $L_2$ ) Gram-Schmidt canceller with noise outliers  
6 present in the weight training data. Thus it is apparent that the pseudo-median ( $L_{med}$ )  
7 cascaded canceller of the present invention has very desirable features compared to  
8 conventional adaptive signal processors.

9       The pseudo-median ( $L_{med}$ ) cascaded canceller of the present invention has an  
10 additional feature which may be used to advantage in adaptive signal processing. Recall  
11 from Fig. 4 that each  $L_{med}$  building block generates a local filtered output signal,  $r$ , by  
12 solving the following equation:  $r = z - w_{med}^* x$ , where  $r = r(k)$ ,  $z = z(k)$ , and  $x = x(k)$ , for  
13  $k = 1, \dots, K$ . Refer next to Fig 10, which is a block diagram of a pseudo-median ( $L_{med}$ )  
14 cascaded canceller of the present invention. Each row of building blocks is designated as  
15  $i$ , where  $i = 1, \dots, N-1$ . In Figure 10, the  $N = 4$  input channel case results in three rows of  
16 building blocks ( $i = 1, 2, 3$ ). The main input channel is designated  $u_1(k)$ . Notice that each  
17 of the left-most building blocks “a first end building block” in each row of Fig 10 receives  
18 a local input signal which is either the main channel  $u_1(k)$  (for the first row of building  
19 blocks) or is derived from the main channel  $u_1(k)$  through prior building blocks (for  
20 building block row  $i=2,3$ ). Also shown in Fig 10, at the right most end of each row of  
21 building blocks, there is a “last end building block”, at the end of the row opposite the first  
22 end building block fed by the main channel  $u_1$ . Each of the  $N-1$  last end building blocks  
23 processes a local auxiliary input fed originally from the from the last ( $N$ th) input channel

1  $(u_N(k))$ . Each of these  $N-1$  last end building blocks has a local filtered output  $r$ , which is  
2 redesignated as  $p_i(k)$  (for  $i = 2, 3, \dots, N$ ) for convenience. There is also a first of these  $p(k)$   
3 values,  $p_1(k)$ , however, which is not an output of a last end  $L_{med}$  building block, but instead  
4 is the  $N$ th input channel (which is an auxiliary channel). So, for the  $N=4$  case in Fig. 10,  
5  $p_1(k) = u_4(k)$ , and generally,  $p_1(k)$  is equal to  $u_N(k)$ .

6 Each of the  $N$  values  $p_i(k)$  ( $i = 1, 2, \dots, N$ ) is approximately statistically uncorrelated  
7 to the rest, and so the  $p_i(k)$  values are uncorrelated with each other. This is a useful form  
8 for input channels to have for input to another processing algorithm (such as for example, a  
9 a Least Mean Square (LMS) algorithm, see Haykin, S. Adaptive Filters Theory, Prentice  
10 Hall, 3<sup>rd</sup> ed., 1996, p 365)). This is a desirable property for data channels that are used as  
11 input channels for follow-on adaptive processors.

12 In this form, the pseudo-median cascaded canceller acts as a robust data channel  
13 pre-processor, and provides the data in a more useful form. Optimally, the  $p_i(k)$  values  
14 which are supplied by the building blocks, are supplied to a second local output channel  
15 that is separate from the local output channel. In this manner, there will be no interference  
16 with the original data flow path of the  $L_{med}$  building block output  $r$ .

17 The invention has been described with reference to certain preferred embodiments  
18 thereof. It will be understood, however, that modification and variations are possible  
19 within the scope of the appended claims.